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RANDOM PROJECTION AND TRUNCATED SVD FOR ESTIMATING DIRECTION OF ARRIVAL IN ANTENNA ARRAY

Introduction. *The need to solve inverse problems arises in many areas of science and technology in connection with the recovery of the object signal based on the results of indirect remote measurements. In the case where the transformation matrix has a high conditional number, the sequence of its singular numbers falls to zero, and the output of the measuring system contains noise, the problem of estimating the input vector is called discrete ill-posed problem (DIP). It is known that the DIP solution using pseudoinverse of the input-output transformation matrix is unstable. To overcome the instability and to improve the accuracy of the solution, regularization methods are used.*

Our approaches to ensuring the stability of the DIP solution (truncated singular decomposition (TSVD) and random projection (RP)) use the integer regularization parameter, which is the number of terms of the linear model. Regularization with an integer parameter makes it possible to provide a model close to the best in terms of the accuracy of the input vector recovery, and also to reduce the computational complexity by reducing the dimensionality of the problem.

The purpose *of the article is to develop an approach to estimating the direction of arrival of signals in the antenna array using the DIP solution, to compare the results with the well-known MUSIC method, to reveal the advantages and disadvantages of the methods.*

Results. *Comparison of TSVD and MUSIC (implemented in real numbers) when working with correlated sources and five snapshots showed the advantage of TSVD in terms of the power of the useful signal* P_{ratio} *by 2.2 times with the number of antenna elements* $K = 15$ *and by 4.7 times with* $K = 90$. *The advantage of TSVD in* P_{ratio} *is by 3.7 times for* $K = 15$ *and by 4.2 times for* $K = 90$. *Comparison of RP and MUSIC (implemented in real numbers), when working with correlated sources and five snapshots, showed the advantage of RP in* P_{ratio} *by 3* *times at K = 15 and by 4.4 times at K = 90. When working with 100 snapshots, the advantage of RP in P_{ratio} is by 3.8 times for K = 15 and by 4.2 times for K = 90.*

Conclusions. *The approach to determining the direction of arrival based on the l_2-regularization methods provides a stable solution in the case of a small number of snapshots, high noise and correlated source signals. Methods of determining the direction of arrival based on l_2-regularization, in contrast to l_1-regularization, do not impose restrictions on the properties of the input-output transformation matrix, do not require a priori information on the number of signal sources, allow constructing efficient hardware implementations.*

Keywords: *Direction of arrival estimation, truncated singular value decomposition, random projection, MUSIC.*

INTRODUCTION

The need to solve inverse problems arises in many areas of science and technology in connection with the recovery of the object signal based on the results of indirect remote measurements. The transformation of the object signal when interacting with the propagation medium and the measuring system is modeled by a linear input-output transformation matrix. The transformation matrix and the vector of the results of indirect measurements (the output vector) are known, it is required to determine the vector of the input signal (the input vector, i.e., the solution vector).

In the case where the transformation matrix has a high conditional number, the sequence of its singular numbers falls to zero, and the output of the measuring system contains noise, the problem of estimating the input vector is called discrete ill-posed problem (DIP) [1, 2]. Discrete ill-posed problems arise, for example, in such areas as spectrometry, gravimetry, magnetometry, electrical prospecting and others [3]. Similar properties are possessed by the matrix formed by the steering vectors of the antenna array.

It is known that the DIP solution using pseudoinverse of the input-output transformation matrix is unstable. Small changes in the measurement (output) vector lead to large changes in the solution vector, while the value of the solution error is large. To overcome the instability and, accordingly, to improve the accuracy of the solution, regularization methods are used. In one of the regularization approaches [1, 2] the functional of the least-squares method is complemented by the restriction of the norm of the parameter vector, weighted by the regularization parameter [2]. The disadvantages of this regularization method are the high computational complexity and complexity of the selection of an adequate value of the regularization parameter, on which the stability and accuracy of the solution largely depends. To overcome the instability and increase the accuracy of the DIP solution, one could use an approach based on Truncated Singular Value Decomposition (TSVD) [4–7] and an approach using random projection [8–16].

In this article, the problem of estimating the direction of arrival (DOA) of signals in an antenna system is considered as a DIP. A linear model for obtaining the output vector from the input vector (from the signal sources) is given, the importance of determining DOA from a small number of output vectors (snapshots or samples) is considered, and a brief review of the methods for the DOA estimation is given. The methods of solving the DIP based on the Truncated Singular Value Decomposition and on the Random Projection and their application for the DOA estimation are considered. Results of the simulation of DOA estimation by these methods and comparison with results obtained by the known MUSIC method [17] are given. Based on the analysis of the experimental study, the domain of applicability of methods for determining the direction of arrival based on the Truncated Singular Value Decomposition and on the Random Projection are given as well as directions of further research.

DIRECTION OF ARRIVAL ESTIMATION IN THE ANTENNA ARRAY

The model of output formation for antenna array. We will model the output of the antenna system under the assumption that the signal sources are distant and narrow-band [18]. Assuming a distant source, the wave on the array of receivers is a plane wave moving from the source to the origin. The vector of the output of the antenna array of K elements in the case of M plane waves incident on it is written as follows:

$$\mathbf{y}(t) = \mathbf{A}(\theta)\,\mathbf{x}(t) + \boldsymbol{\varepsilon}(t),$$

where $\mathbf{A}(\theta)$ is the matrix $K \times M$, formed by the antenna array steering vectors $\{\mathbf{a}(\theta_i)\}$, $i = 1,\ldots, M$, $\mathbf{x}(t)$ is the source signal vector (of dimension M), $\boldsymbol{\varepsilon}(t)$ is a K-component white noise vector, t is the time moment. The vector $\mathbf{y}$ obtained at a specific time will also be called a sample (also known as snapshot). The total number of samples will be denoted by N.

Elements of the vector $\mathbf{a}(\theta_i)$ are determined by the phase of the i-th signal (the signal received from the i-th direction) onto the corresponding antenna element:

$$\mathbf{a}(\theta_i) = [e^{-j\frac{2\pi}{\lambda}d_1 \sin(\theta_i)}, e^{-j\frac{2\pi}{\lambda}d_2 \sin(\theta_i)}, \ldots, e^{-j\frac{2\pi}{\lambda}d_K \sin(\theta_i)}]^{\mathrm{T}},$$

where λ is the carrier wavelength, $d_k = (k - 1)d$, d is the distance between the antenna elements (Fig. 1).

The task is to estimate the direction of arrival of the signals, using the information contained in $\mathbf{y}(t)$ and the known matrix $\mathbf{A}(\theta)$.

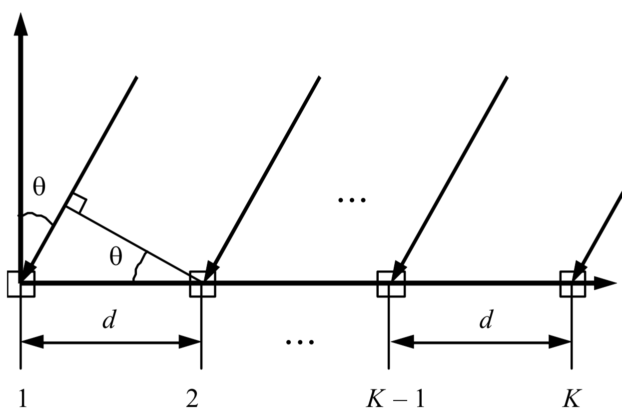


Fig. 1. The scheme of receiving a plane wave from a remote source by a linear uniform antenna array

Methods for estimating the direction of arrival of a signal using an array of antennas. Classical methods of processing data from antenna array can be divided into parametric and nonparametric methods. The first group includes, for example, methods based on the maximum likelihood principle. And the second group includes methods based on spectral approaches.

The methods based on the maximum likelihood, in turn, can be divided into deterministic and stochastic ones, where the signals are considered, respectively, as deterministic or stochastic processes. In the process of solving the optimization problem, the parameters corresponding to the arrival directions of the signal are chosen so as to maximize the likelihood function. Parametric approaches provide an accurate estimate of the DOA, but have a high computational complexity.

Nonparametric methods for DOA estimating can be divided into two main subgroups: methods of beam forming and methods based on subspaces. Nonparametric methods have less computational complexity with respect to parametric ones.

The popularity of approaches to the DOA estimation on the basis of subspaces is explained by the fact that they provide high resolution. The representative of the methods of this group is MUSIC [17–19]. MUSIC uses the Eigen Value Decomposition (EVD) of the covariance matrix of samples (or snapshots, i.e., signals received by the antenna system), on the basis of which the signal and noise subspaces are formed. The signal subspace is formed by selecting M eigenvectors (spanned by M eigenvectors), where M is the number of signal sources. The noise subspace is spanned by $K-M$ eigenvectors. Further analysis and obtaining power spectrum is based on the orthogonality of the signal and noise subspaces.

MUSIC has a higher performance than conventional beamforming [20]. In the case where the number of data samples obtained from each element of the antenna array is sufficiently large, the method provides statistically consistent estimates.

One of the drawbacks of MUSIC appears in the situation when some of the source signals are strongly correlated, the efficiency of the algorithm in this case deteriorates sharply. In this case, the number of large eigenvalues becomes less than the number of signals. Another disadvantage is the reduced efficiency of MUSIC with a small number of samples.

In the framework of parametric methods for DOA estimating, methods based on sparse representation and l_1-regularization, have been developed. The use of the concept of sparsity for DOA estimation was first proposed in [21, 22], where the source localization problem was posed as a linear inverse problem and the l_1-SVD method was developed. The formulation of the task of estimating DOA using sparseness is natural in the sense that the number of directions from which signals are sent to the antenna array is much smaller than the total number of directions analyzed when processing data from the antenna array. Steering vectors for all possible spatial positions where a source can exist, form a so-called "overcomplete dictionary".

The l_1-regularization methods estimate the DOA based on the data from the array output and the overcomplete dictionary [23, 24].

Approaches based on sparsity can provide high resolution when working with a limited amount of noisy data and with correlated signal sources. The drawbacks of this approach include strict requirements to the properties of the overcomplete dictionary, i.e. to the value of its mutual coherence. To obtain reliable estimates based on this approach, one needs to increase the number of antenna elements. The drawbacks also include the complexity of hardware implementation of approaches based on sparsity.

Therefore, the study of methods for solving the problem of the DOA estimation based on l_2-regularization methods deserves attention. This approach provides a stable solution in the case of a small number of samples (snapshots), high noise and correlated source signals. At the same time, it does not impose restrictions on the properties of matrix **A**, and also allows building effective hardware implementations. The advantages of approaches using regularization include work in the absence of a priori information on the number of signal sources (which is required, for example, in MUSIC).

REGULARIZATION METHODS FOR SOLVING DIP

Consider a signal transformation described by a linear model of the form

$$\mathbf{y} = \mathbf{A}\mathbf{x} + \boldsymbol{\varepsilon}, \tag{1}$$

where the matrix $\mathbf{A} \in \Re^{K \times L}$ and the measurement vector $\mathbf{y} \in \Re^{K}$ ($\mathbf{y} = \mathbf{y}_0 + \boldsymbol{\varepsilon}, \mathbf{y}_0 = \mathbf{A}\mathbf{x}$) are known. The components of the noise vector $\boldsymbol{\varepsilon} \in \Re^{K}$ are realizations of independent, normally distributed random variables with zero mean and σ^2 variance. It is required to estimate the signal vector $\mathbf{x} \in \Re^{L}$. With respect to the antenna array, L is the number of directions, K is the number of antenna elements.

In the case when **y** contains noise and sequence of singular values of the **A** matrix tends to zero (and **A** has a large condition number), the estimation problem is called a discrete ill-posed problem. For DIP, the solution (signal vector estimation) obtained by pseudo-inversion as $\mathbf{x}^* = \mathbf{A}^{+}\mathbf{y}$, where $\mathbf{A}^{+}$ is the pseudo-inverse matrix, is unstable and inaccurate. To overcome the instability and improve the accuracy of the solution, the regularization approach is used.

The classical regularization method is the regularization of Tikhonov. The Tikhonov regularization problem of the standard form is formulated as follows:

$$\mathbf{x}_{reg} = \underset{\mathbf{x}}{arg\,min}\left(\|\mathbf{A}\mathbf{x} - \mathbf{y}\|_2 + \lambda\|\mathbf{x}\|_2\right), \tag{2}$$

where λ is the regularization parameter. To select the regularization parameter, special methods are used, such as the L-curve method, the generalized discrepancy method, the cross-validation method [1]. Methods for solving DIP based on Tikhonov regularization are inherent in such shortcomings as the difficulty in correctly selecting the regularization parameter and the computational complexity.

Another approach to ensuring the stability of DIP solution uses an integer regularization parameter, which is the number of terms of the model that approximates the initial data (the model is linear with respect to parameters). To obtain a stable solution (the estimate **x***), for example, methods such as truncated singular value decomposition [1], truncated QR decomposition [25, 14], a method based on randomization [7, 10, 16] can be used.

The vector **x*** that estimates **x** based on the truncated singular value decomposition is obtained by the following linear model [1]:

$$\mathbf{x}_{k\ SVD}^{*} = \mathbf{V}_k \mathbf{S}_k^{-1} \mathbf{U}_k^{\mathrm{T}} \mathbf{y} = \sum_{i=1}^{k} \mathbf{v}_i s_i^{-1} \mathbf{u}_i^{\mathrm{T}} \mathbf{y}\,, \tag{3}$$

where **u** are left singular vectors, **v** are right singular vectors, s are singular values.

The estimation vector on the basis of random projection is obtained by a linear model of the form [7]

$$\mathbf{x}_{k\ RP}^{*} = (\mathbf{R}_k \mathbf{A})^{+} \mathbf{R}_k \mathbf{y} = \sum_{i=1}^{k} \mathbf{c}_i \mathbf{r}_i^{\mathrm{T}} \mathbf{y}\,, \tag{4}$$

where **r** is the column of the random matrix $\mathbf{R}_k$, **c** is the column of the matrix $(\mathbf{R}_k\mathbf{A})^{+}$.

There is an optimal number k of terms of the linear models (3) and (4), which minimizes the average recovery error of **x**

$$e_x(k) = \mathrm{E}\{\|\mathbf{x} - \mathbf{x}_k^{*}\|^2\}\,, \tag{5}$$

where $\mathrm{E}\{\cdot\}$ is the expectation operator over the noise realizations in the measurement vector, $\mathbf{x}_k^*$ is the vector of the recovered signal, i.e. the vector of **x** estimate by (3) or (4).

Existence of optimal $k < N$ is possibly due to the fact that the error of the true signal reconstruction can be represented as a sum of two terms, one of which (deterministic) decreases with increasing of the model component number, and the other (stochastic) term increases and is proportional to the noise level in the measurement vector [16, 7, 10]. Thus, at a certain noise level, the global minimum of error can be achieved at $1 < k < N$.

Unlike regularization associated with the minimization of the functional (2), where the regularization parameter is a real number, the approach to solving DIP by regularization with an integer parameter makes it possible to ensure selection of the best model in terms of error (accuracy) of **x** recovery (5), by enumerating N of k-component models (3), (4). Moreover, the solution of DIP based on random projection (4) makes it possible to reduce the computational complexity by reducing the dimensionality of the problem when $k < N$ [16].

Solution of DIP based on singular value decomposition. The solution of DIP based on singular value decomposition is obtained [25, 1] as follows:

$$\mathbf{x}^* = \mathbf{A}_k^{+}\mathbf{y}\,,\ \mathbf{A}_k^{+} = \mathbf{V}\mathbf{S}^{-1}\mathbf{U}^{\mathrm{T}}\,.$$

Here $\mathbf{A}_k$ is the **A** matrix approximation obtained from the k components of SVD, **U** is the matrix of left singular vectors with orthonormal columns, **V** is the matrix of right singular vectors with orthonormal columns, **S** is the matrix of singular values, and $\mathbf{A}^{+}$ is the pseudo-inversion of **A**. The estimation of the solution **x***

$$\mathbf{x}^* = \sum_{i=1}^{k} \frac{\mathbf{u}_i^{\mathrm{T}}\mathbf{y}}{s_i}\mathbf{v}_i$$

is formed as the sum of the vectors $\mathbf{v}_i$ weighted by the coefficients $w_i = \langle \mathbf{u}_i, \mathbf{y}\rangle / s_i$. As the index i increases, the singular vectors become more and more alternating, noise-like. In the case where the estimate **x*** is determined primarily by the terms of the sum corresponding to large singular values (that is, to smooth singular vectors), a smoothness and a small error of the solution are ensured. If **x*** is determined by the terms of the sum corresponding to small singular values (strongly alternating singular vectors) the solution error increases. It is intuitively clear that there can be some optimal number of components in expression (3) that is sufficient to deliver all the features of the modeled signal, but does not include noise-like singular vectors.

In [5–7] an approach was developed to determine the optimal number of components of the SVD for the solution of DIP, that is, such number that the accuracy of the solution is maximal. Let us consider the accuracy of the recovery of the true signal **x** and the accuracy of the recovery of the output vector **y** when solving DIP on the basis of SVD.

In [5–7], an expression was obtained for the RMS error of **x** recovery

$$e_x(k) = \left\|(\mathbf{A}_k^+\mathbf{A}_k - \mathbf{I})\mathbf{x}\right\|^2 + \sigma^2 \mathrm{trace}(\mathbf{A}_k^{+\mathrm{T}}\mathbf{A}_k^+) \tag{6}$$

and its components

$$e_\mathrm{d}(k) = \left\|(\mathbf{A}_k^+\mathbf{A}_k - \mathbf{I})\mathbf{x}\right\|^2 \text{ and } e_\mathrm{s}(k) = \sigma^2 \mathrm{trace}(\mathbf{A}_k^{+\mathrm{T}}\mathbf{A}_k^+),$$

where e_d is the deterministic component of the recovery error and e_s is the stochastic component of the error.

The expression for the output recovery error [5–7] has the form

$$e_y(k) = \left\|(\mathbf{A}_k\mathbf{A}_k^+ - \mathbf{I})\mathbf{y}_0\right\|^2 + \sigma^2 \mathrm{trace}(\mathbf{A}_k^{+\mathrm{T}}\mathbf{A}_k^\mathrm{T}\mathbf{A}_k\mathbf{A}_k^+). \tag{7}$$

Components of the output recovery error:

$$e_\mathrm{d}(k) = \left\|(\mathbf{A}_k\mathbf{A}_k^+ - \mathbf{I})\mathbf{y}_0\right\|^2,\ e_\mathrm{s}(k) = \sigma^2 \mathrm{trace}(\mathbf{A}_k^{+\mathrm{T}}\mathbf{A}_k^\mathrm{T}\mathbf{A}_k\mathbf{A}_k^+),$$

where e_d is the deterministic component of the recovery error and e_s is the stochastic component of the error.

In practice, it is impossible to calculate the recovery error due to the lack of information about **x**, therefore it is impossible to determine the optimal k directly by (6). For a choice of k close to optimal, the model selection criteria have been developed (i.e., functions having an extremum at k close to or equal to the optimal k).

Experimental studies [16, 7] have shown that there an optimal number of the components of linear models (3) and (4) exists, that minimizes the error (5). The optimum exists because the true signal recovery error can be represented as a sum of two components, one of which (deterministic) decreases with the increasing number of model components (the model dimensionality), and the other (stochastic) grows and is proportional to the noise level in the measurement vector [15, 16, 10, 7]:

$$e(k) = e_\mathrm{d}(k) + e_\mathrm{s}(k) = e_\mathrm{d}(k) + \sigma^2 e_\mathrm{g}(k), \tag{8}$$

where $e_\mathrm{d}(k)$ is the value of the deterministic error component for the model of dimensionality k, $e_\mathrm{s}(k)$ is the value of the stochastic error component for the model of dimensionality k, $e_g(k) = e_s(k)/\sigma^2$. Thus, at a certain noise level, the global minimum of the error can be achieved at $1 \le k \le N$.

The representation of the error in the form (10), the study of the error components and the development of the model selection criteria (MSC) are the techniques used by the inductive modelling approach [26–28] to find the optimal solution. In practice, it is impossible to calculate the recovery error $e_x(k)$ due to the lack of information about **x**, therefore it is impossible to determine the optimal k directly. For the choice of k close to the optimal, MSC is used, that is, a function that would have an extremum at k close to or equal to the optimal one.

Solution of DIP using random projection. One of the problems of using SVD-decomposition for the solution of DIP is its high computational complexity $O(N^3)$ (for a square matrix). The approach [15, 8, 16, 10, 7] based on finding the minimum error of solving a discrete ill-posed problem using random projection, ensures the stability of the solution and allows one to reduce the computational complexity. Random projections and other randomizations are also used for various versions of DIPs in [29–32].

Random projection is a kind of methods for the formation of neural network distributed representations. Distribution representations include not only random projection based methods [33–35], but also a number of other representation schemes for vectors, such as those based on receptive fields [36] or compositional methods [37–41] as well as for structured data, e.g. [42–48]. Note that distributed representations are closely related to associative memory, e.g. [49, 50] as well as to human memory [51].

To find solution on the basis of the random projection approach, we multiply both parts of the original equation (1) by a matrix **Q** and obtain the (approximate) equation:

$$\mathbf{F}_k \mathbf{x} = \mathbf{b}_k \text{, where, } \mathbf{F}_k = \mathbf{Q}_k^{\mathrm{T}} \mathbf{A},\ \mathbf{F}_k \in \Re^{k \times N},\ \mathbf{b}_k = \mathbf{Q}_k^{\mathrm{T}} \mathbf{y},\ \mathbf{b}_k \in \Re^k .$$

It was proposed in [16] to obtain the matrix **Q** by the QR-decomposition of the matrix **GA** = **QR**, where **Q** is the orthonormal matrix, **R** is upper triangular matrix. Elements of the matrix **G** are the realization of a random variable with a normal distribution, zero mean and unit variance, $k \le N$.

Recovery of **x** based on the pseudoinversion is obtained as

$$\mathbf{x}_Q^* = \mathbf{F}_k^+ \mathbf{b}_k .$$

In [16], the expression for the mean-squared error of the **x** recovery was obtained for the random projection method:

$$e_x = \left\| (\mathbf{F}_k^+ \mathbf{F}_k - \mathbf{I})\mathbf{x} \right\|^2 + \sigma^2 \mathrm{trace}(\mathbf{F}_k^{+\mathrm{T}} \mathbf{F}_k^+)$$

and its components are:

$$e_{\mathrm{d}} = \left\| (\mathbf{F}_k^+ \mathbf{F}_k - \mathbf{I})\mathbf{x} \right\|^2,\ e_{\mathrm{s}} = \sigma^2 \mathrm{trace}(\mathbf{F}_k^{+\mathrm{T}} \mathbf{F}_k^+),$$

where e_{d} is the deterministic component of the recovery error and e_{s} is the stochastic component of the error.

The expression for the mean-square error of $\mathbf{y}_0$ recovery has the form:

$$e_y = \left\|(\mathbf{AF}_k^{+}\mathbf{Q}_k^{\mathrm{T}} - \mathbf{I})\mathbf{y}_0\right\|^2 + \sigma^2 \mathrm{trace}(\mathbf{F}_k^{+\mathrm{T}}\mathbf{A}^{\mathrm{T}}\mathbf{AF}_k^{+}).$$

Components of $\mathbf{y}_0$ recovery errors are:

$$e_{y\,\mathrm{d}} = \left\|(\mathbf{AF}_k^{+}\mathbf{Q}_k^{\mathrm{T}} - \mathbf{I})\mathbf{y}_0\right\|^2, \; e_{y\,\mathrm{s}} = \sigma^2 \mathrm{trace}(\mathbf{F}_k^{+\mathrm{T}}\mathbf{A}^{\mathrm{T}}\mathbf{AF}_k^{+}),$$

where e_{d} is the deterministic component of the recovery error of the measurement vector and e_{s} is its stochastic component.

EXPERIMENTAL STUDY OF THE DOA ESTIMATION METHODS BASED ON REGULARIZATION

Let us consider the results of simulation of DOA estimation using the methods of truncated singular value decomposition (TSVD), random projection (RP), and MUSIC.

From the above theoretical analysis it follows that the advantages of the TSVD and the RP methods (as parametric methods) with respect to the non-parametric MUSIC are most pronounced when working with a small number of samples (in the limit with single sample) and also in the case of correlating source signals. The experimental study was aimed at experimentally confirming the conclusions about the conditions for the best performance of TSVD and RP.

We used important characteristics of the methods for the DOA estimation. The first characteristic is the dependence of the output power ("spatial power") on the angle $P(\theta)$ that shows how the power in signal direction exceeds the non-signal power. The second one is the ratio of the maximum power value outside the directions of the source signals to the maximum power value in the directions of the source signals that we denote as P_{ratio}.

We used linear array antenna with element spacing $d = \lambda/2$, the carrier wavelength λ was equal to 150.

Simulation modeling was carried out for the case of two signal sources (sin waves) with angular coordinates of 10 and 20 degrees and with a signal-to-noise ratio SNR = 0 (Gaussian noise).

We experimentally investigated non-correlated and correlated source signals, various number of antenna elements, and various number N of snapshots and measured the dependences $P(\theta)$ and $P_{\mathrm{ratio}}(K)$. The non-correlated sources were modelled by using different (low) frequencies $\omega_1 = \pi/4$, $\omega_2 = \pi/3$ for the baseband complex signal (known as complex envelope). The correlated sources were modelled by using the same frequency $\omega_1 = \pi/4$, $\omega_2 = \pi/4$.

Study of MUSIC and TSVD in complex numbers. Let's compare MUSIC and TSVD in the case of two signal sources of 10 and 20 degrees with the frequencies $\omega_1 = \pi/4$, $\omega_2 = \pi/3$, respectively, at signal-to-noise ratio SNR = 0.

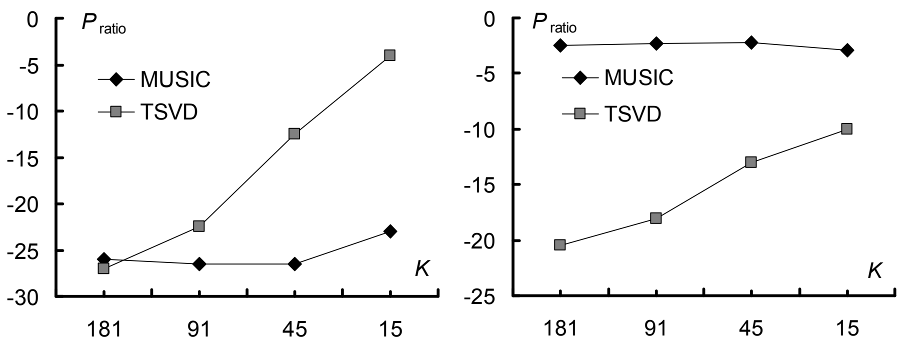
We will calculate P_{ratio} for antenna arrays with different number of elements: $K = \{15, 45, 91, 181\}$.

P_{ratio} dependence on the number of antenna elements was investigated for the number of samples $N = 100$ and $N = 1$. The results are shown in Fig. 2. With the number of samples $N = 100$, the MUSIC method provides a stably high P_{ratio} value at –26, –26.5 dB for $K = \{45, 91, 181\}$ and –23 dB for $K = 15$. P_{ratio} for the TSVD method with $N = 100$ improves with an increase in the number of antenna elements from –4 dB at $K = 15$ to –27 dB at $K = 181$.

Conversely, in the case of single sample ($N = 1$), the MUSIC method shows a small value of P_{ratio} at –2.5, –2.9 dB. The TSVD method is superior to the MUSIC for all values of K. The suppression for the TSVD method with $N = 1$ improves with increasing K, from –10 dB at $K = 15$ to –20.5 dB at $K = 181$.

Examples of the $P(\theta)$ dependence for the MUSIC and TSVD methods for $K = \{180, 15\}$ are shown in Fig. 3, 4. The range of angles from –90 to 90 degrees.

For the TSVD method (at $K = 180$, $N = 100$, $\omega_1 = \pi/4$, $\omega_2 = \pi/3$), the $P(\theta)$ dependence (Fig.3) outside the directions to the sources has a constant (of the order of –32 dB) level in the range of angles from –45 to 45 degrees. In the range of angles –45... –90 and 45...90 degrees, the $P(\theta)$ dependence gradually decreases to –50 dB.



a) $N = 100$ b) $N = 1$

Fig. 2. Dependence of P_{ratio} on the number K of antenna elements (MUSIC and TSVD in complex numbers)

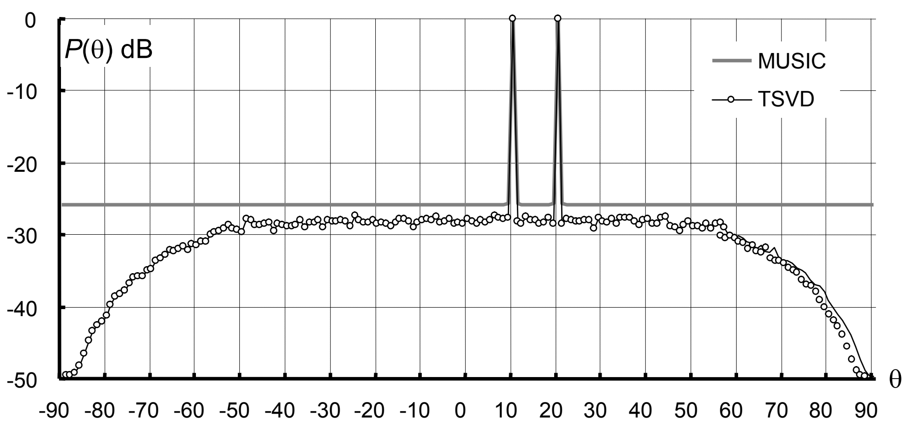


Fig. 3. Dependence $P(\theta)$ for MUSIC and TSVD methods at $K = 180$, $N = 100$, SNR = 0, $\omega_1 = \pi/4$, $\omega_2 = \pi/3$

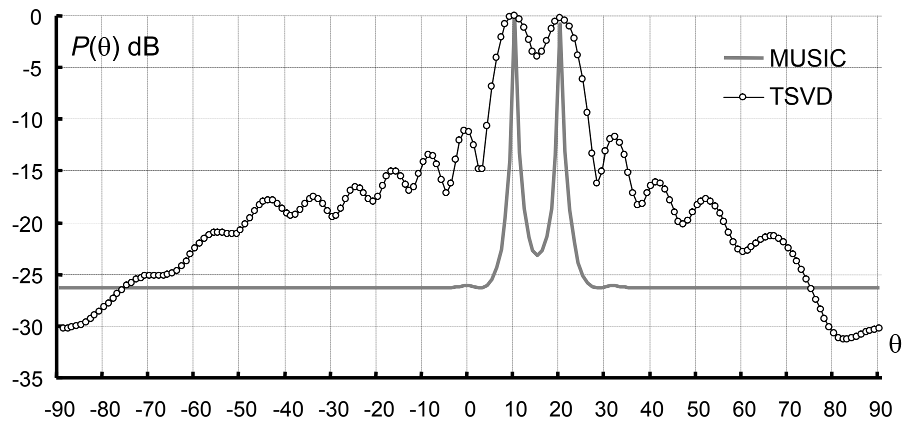


Fig. 4. Dependence $P(\theta)$ for MUSIC and TSVD methods at $K = 15$, $N = 100$, SNR = 0, $\omega_1 = \pi/4$, $\omega_2 = \pi/3$ (in complex numbers)

The peaks in the directions to the signal sources are narrow, the area between the peaks is flat without spikes, which makes it easy to identify the directions to the signal sources. The MUSIC method outside the directions to the signal sources provides $P(\theta)$ at –35 dB across the entire range of angles (–90 to 90 degrees) and well-defined peaks in the directions to the signal sources. Thus, with the above parameters, both methods (TSVD and MUSIC) effectively solve the problem of determining the direction of arrival of the signal, and the TSVD works somewhat better than MUSIC.

For the MUSIC method (at $K = 180$, $N = 100$, $\omega_1 = \pi/4$, $\omega_2 = \pi/3$), the $P(\theta)$ dependence (Fig.4) has a constant level of –27 dB outside the directions to the sources. Peaks in the directions to the sources are well pronounced, but they have a broadening at the base up to 10 degrees. For the TSVD method, $P(\theta)$ has a "lobe" character, and it is possible to determine the direction of arrival as the maxima of $P(\theta)$ if one selects an adequate threshold value for $P(\theta)$. In this experiment, the MUSIC method works better than TSVD.

For the TSVD method (at $K = 180$, $N = 1$, $\omega_1 = \pi/4$, $\omega_2 = \pi/3$), the peaks in the directions to the sources are well defined (narrow), the $P(\theta)$ dependence (Fig.5) outside the directions to the sources has a noise-like character with a constant average (of the order of –32 dB in the range of angles from –50 to 50 degrees). For the MUSIC method, outside the directions to the sources $P(\theta)$ in the entire range of angles (from –90 to 90 degrees) is at the level of –2.5 dB. There are peaks in the directions to the signal sources. In this example, TSVD solves the problem of determining the direction of arrival of a signal much better than MUSIC.

For the experiment presented in Fig. 6 (at $K = 15$, $N = 1$, $\omega_1 = \pi/4$, $\omega_2 = \pi/3$), the $P(\theta)$ dependence obtained by the TSVD method has a "lobe" character, $P(\theta)$ for a maximum lateral lobe is of –10 dB. The $P(\theta)$ dependence obtained by the MUSIC method also has a number of false peaks, the maximum level of which is –3 dB. In this experiment, the TSVD method works better than MUSIC.

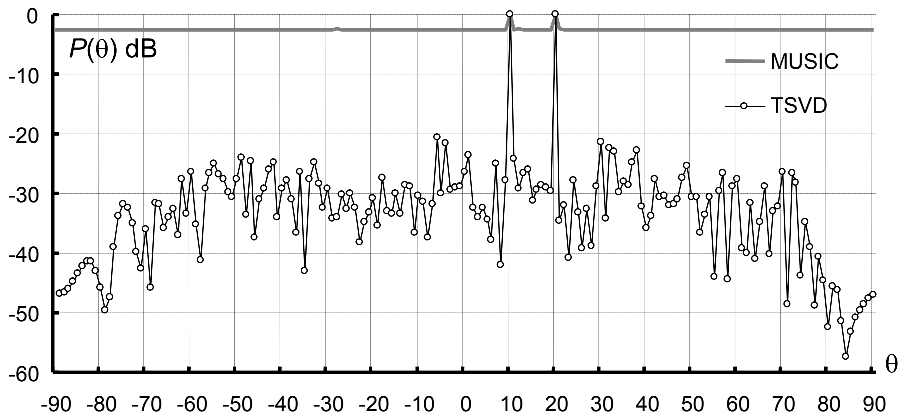


Fig. 5. Dependence $P(\theta)$ for MUSIC and TSVD methods at $K = 180$, $N = 1$, SNR = 0, $\omega_1 = \pi/4$, $\omega_2 = \pi/3$ (in complex numbers)

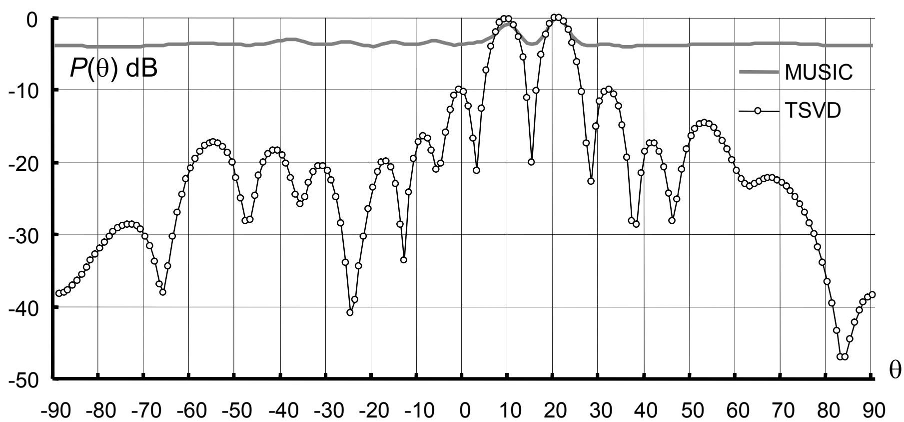


Fig. 6. Dependence of P(θ) for MUSIC and TSVD methods at $K = 15$, $N = 1$, SNR = 0, $\omega_1 = \pi/4$, $\omega_2 = \pi/3$ (in complex numbers)

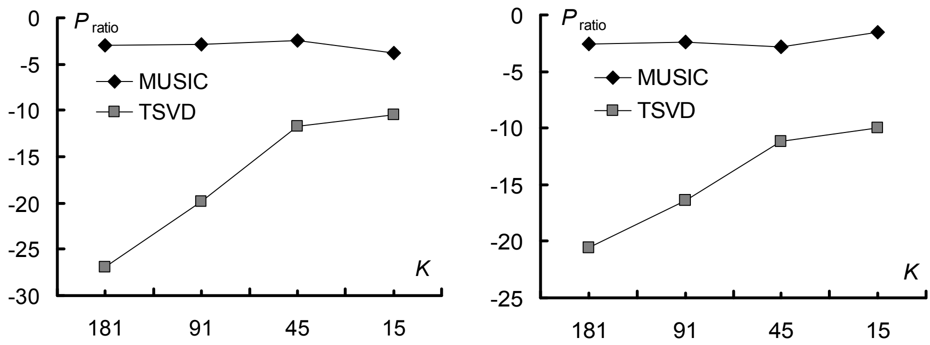
In many practical tasks it is necessary to know the angular coordinates of the signal sources that (are correlated, i.e.) have the same frequency. For the experiment, we form the signal sources with $\omega_1 = \omega_2 = \pi/4$.

We obtain the dependence (Fig.7) of P_{ratio} on the number K of antenna elements for the number of samples $N = 100$ and $N = 1$.

In the case $\omega_1 = \omega_2$, the MUSIC method demonstrates a poor P_{ratio} both for $N = 100$ (at the level of –2.46, –3.75 dB) and for $N = 1$ (–1.5, –2.8 dB).

For the TSVD method, the P_{ratio} value at $N = 100$ improves with an increase in the number K of antenna elements from –10.5 dB at $K = 15$ to –26.88 dB at $K = 181$ and at $N = 1$: –10 dB to –20.6 dB. This is much better than for the MUSIC method.

Examples of the $P(\theta)$ dependence for MUSIC and TSVD for $K = \{181, 15\}$ are shown in Fig. 8, 9.



a) $N = 100$ b) $N = 1$

Fig. 7. Dependence of P_{ratio} on the number K of antenna elements in the case $\omega_1 = \omega_2$ (MUSIC and TSVD in complex numbers)

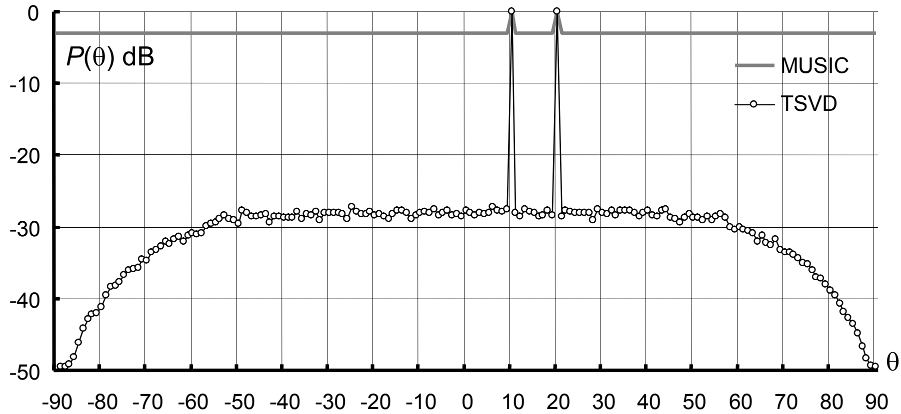


Fig. 8. Dependence $P(\theta)$ for MUSIC and TSVD methods at $K = 180$, $N = 100$, SNR = 0, $\omega_1 = \pi/4$, $\omega_2 = \pi/4$ (in complex numbers)

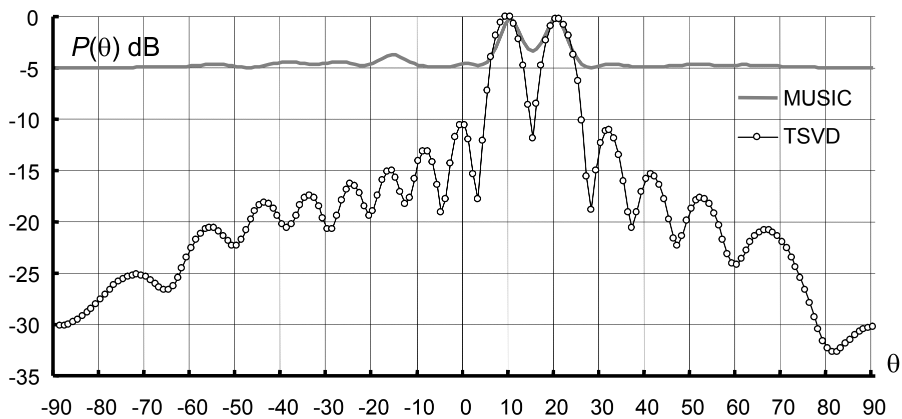


Fig. 9. Dependence $P(\theta)$ for MUSIC and TSVD methods at $K = 15$, $N = 100$, SNR = 0, $\omega_1 = \pi/4$, $\omega_2 = \pi/4$ (in complex numbers)

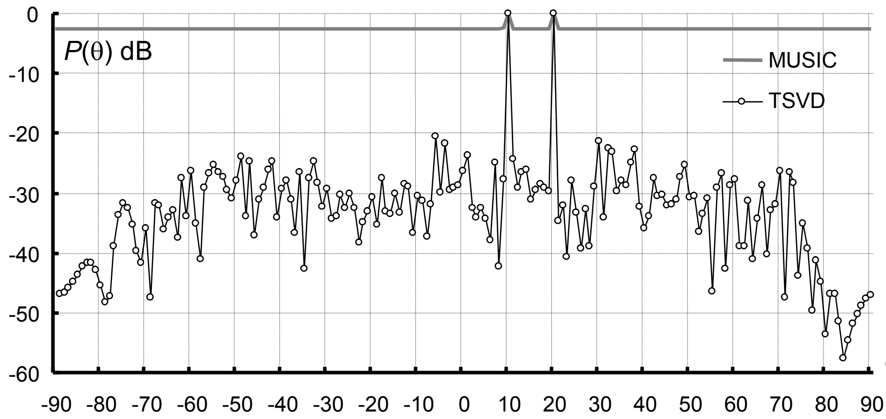


Fig. 10. Dependence $P(\theta)$ for MUSIC and TSVD methods at K = 180, N = 1, SNR = 0, ω1 = π/4, ω2 = π/4 (in complex numbers).

The $P(\theta)$ dependences (at $K = 180$, $N = 100$, $\omega_1 = \pi/4$, $\omega_2 = \pi/4$ (Fig. 8) and at $K = 15$, $N = 100$, $\omega_1 = \pi/4$, $\omega_2 = \pi/4$ (Fig. 9)) are similar to those of $P(\theta)$ in Fig. 3 and Fig. 4. For TSVD outside the directions to the sources (Fig. 8), $P(\theta)$ has a constant level of –28 dB (from –45 to 45 degrees) and outside drops to –50 dB. For MUSIC, $P(\theta)$ is at –3 dB outside the directions to the sources. That is, at $\omega_1 = \omega_2$, the MUSIC method works much worse than TSVD, and a sufficiently large number of samples ($N = 100$) does not improve the situation. For TSVD outside the directions to the sources (Fig.9), $P(\theta)$ has a "lobe" character, the maximum side lobe level is –11 dB. For MUSIC, $P(\theta)$ outside the directions to the sources is at –5 dB, the maximum false peak level is –3 dB; which is much worse than for TSVD.

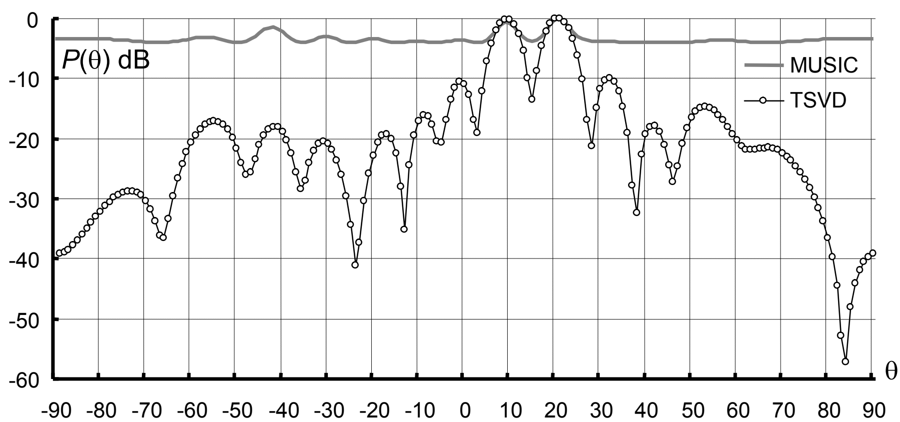


Fig. 11. Dependence $P(\theta)$ for MUSIC and TSVD methods at $K = 15$, $N = 1$, SNR=0, $\omega_1 = \pi/4$, $\omega_2 = \pi/4$ (in complex numbers)

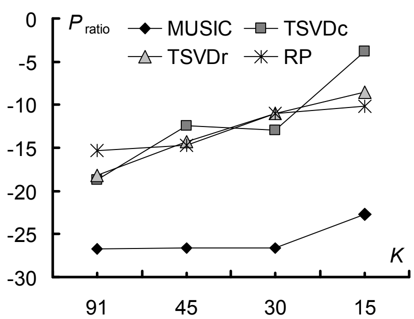
The $P(\theta)$ dependences (at $K = 181$, $N = 100$, $\omega_1 = \pi/4$, $\omega_2 = \pi/4$ (Fig. 10) and at $K = 15$, $N = 100$, $\omega_1 = \pi/4$, $\omega_2 = \pi/4$ (Fig. 11)) are similar to those of $P(\theta)$ in Fig. 5 and Fig.6. For TSVD outside the directions to the sources (Fig. 10), $P(\theta)$ has a noise-like character with a constant level of –33 dB (from –45 to 45 degrees) and outside drops to –48 dB. For MUSIC, $P(\theta)$ is at –3 dB outside the directions to the sources. That is, at $\omega_1 = \omega_2$ and $N = 1$, the MUSIC method works much worse than TSVD. For TSVD outside the directions to the sources (Fig. 11), $P(\theta)$ has a "lobe" character, the maximum side lobe level is –10 dB. For MUSIC, $P(\theta)$ outside the directions to the sources is at –4 dB, the maximum false peak level is –2 dB, which is much worse than for TSVD.

Study of TSVD and RP in real numbers. Let us compare the methods based on the random projection (RP) vs TSVD and MUSIC in the case of two signal sources of 10 and 20 degrees with the frequencies ω_1, ω_2 (Fig. 12) and with the signal-to-noise ratio SNR = 0. The range of angles is from 0 to 90 degrees. The simulation for RP was carried out in real numbers.

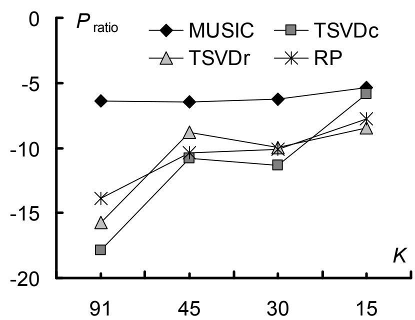
In the case of $\omega_1 = \omega_2$ (Fig. 13), MUSIC demonstrates a poor P_{ratio} for both $N = 100$ (at the level of –2.35, –4.3 dB), and for $N = 5$ (–2.7, –3.26 dB). The P_{ratio} value for TSVD (in complex numbers) at $N = 100$ improves with an increase in the number K of antenna elements from –10.2 dB at $K = 15$ to –19.7 dB at $K = 91$ and at $N = 5$ from –10.9 dB to –16.9 dB.

TSVD in real numbers at $N = 100$, an increase in the number of antenna elements leads to an improvement in the P_{ratio} from –8.8 dB at $K = 15$ to –18.1 dB at $K = 91$ and at $N = 5$ from –6.1 dB to –15.6 dB. The P_{ratio} value for RP in real numbers at $N = 100$ improves with the increase in the number of antenna elements from –9.1 dB at $K = 15$ to –17.9 dB at $K = 91$ and at $N = 5$ from –8.0 dB to –14.2 dB. TSVD (both in complex numbers and in real numbers) and RP provide a much better P_{ratio} value than MUSIC. Examples of $P(\theta)$ dependencies for the MUSIC, TSVD, and RP methods for $K = \{91, 30\}$ are shown in Fig.14, 15.

We also investigated the dependence $P(\theta)$ at $K = 91$, $N = 5$, $\omega_1 = \pi/4$, $\omega_2 = \pi/4$ (Fig.14) and at $K = 30$, $N = 5$, $\omega_1 = \pi/4$, $\omega_2 = \pi/4$ (Fig.15) for the MUSIC and TSVDc methods working in complex numbers, as well as for TSVDr and RP methods working in real numbers.

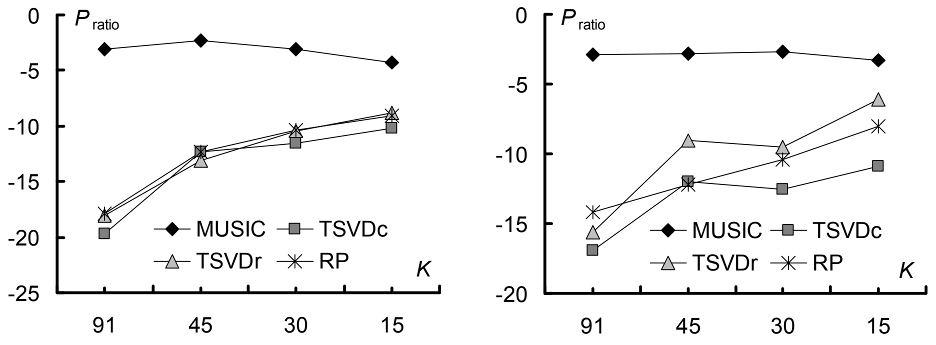


a) $N = 100$



b) $N = 5$

Fig. 12. P_{ratio} dependence on the number K of antenna elements for RP, TSVD and MUSIC (MUSIC and TSVDc in complex numbers, TSVDr and RP in real numbers)



a) $N = 100$ b) $N = 5$

Fig. 13. P_{ratio} dependence on the number K of antenna elements in the case of $\omega_1 = \omega_2$ (MUSIC and TSVDc in complex numbers, TSVDr and RP in real numbers)

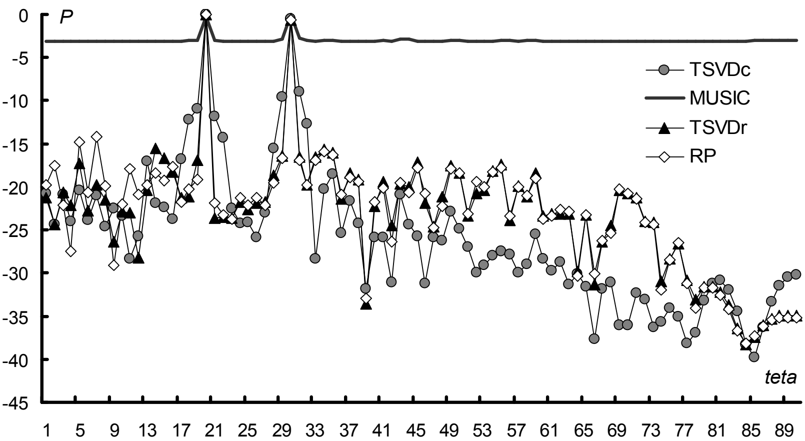


Fig. 14. Dependence $P(\theta)$ for MUSIC, TSVD and RP at $N = 5$, $K = 91$ (MUSIC and TSVDc in complex numbers, TSVDr and RP in real numbers)

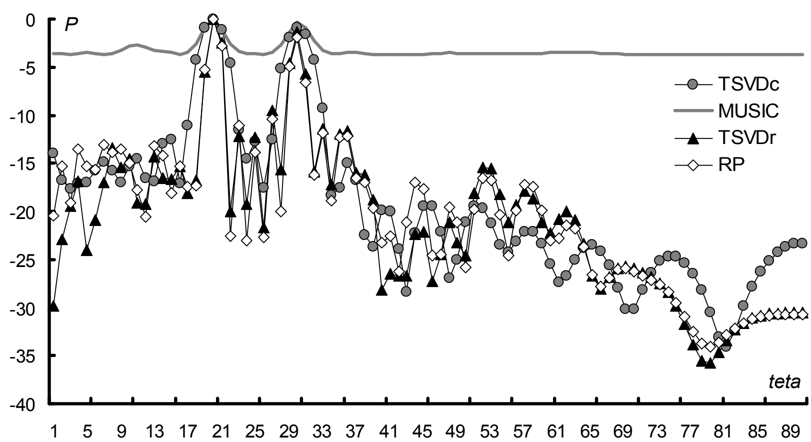


Fig. 15. Dependence $P(\theta)$ for MUSIC, TSVD and RP at $N = 5$, $K = 30$ (MUSIC and TSVDc in complex numbers, TSVDr and RP in real numbers)

The MUSIC method works poorly for both $K = 91$ $P(\theta)$ outside the directions to the sources is –4 dB, and for $K = 30$ $P(\theta)$ outside the directions to the sources is –3.5 dB.

The TSVD methods (in complex and real numbers) and RP work in a similar way to each other. The dependence of $P(\theta)$ outside the directions to the sources has a stochastic (nonsmooth) character and a decreasing trend from –20 dB to –40 dB at $K = 91$, and from –15 dB to –35 dB at $K = 30$.

For TSVDc, $P(\theta)$ outside the directions to the sources at $K = 91$ decreases somewhat faster than for TSVDr and RP; the $P(\theta)$ dependences for TSVDr and RP practically coincide. The entire group of the TSVDc, the TSVDr and the RP methods works better than MUSIC.

CONCLUSIONS

The approach to DOA estimation based on the l_2-regularization methods provides a stable solution in the case of a small number of samples (snapshots), high noise level and correlated source signals.

Comparison of TSVD and MUSIC implemented in complex numbers, under conditions of correlated sources and single sample showed the advantage of TSVD in terms of P_{ratio} by 6.7 times with $K = 15$ and by 7.5 times with $K = 181$. At 100 samples, the advantage of TSVD in terms of P_{ratio} is: by 4.3 times for $K = 15$ and by 7.2 times for $K = 181$.

Comparison of TSVD and MUSIC implemented in real numbers, under conditions of correlated sources, and five samples showed TSVD advantage in terms of P_{ratio} by 2.2 times for $K = 15$ and by 4.7 times for $K = 90$. At 100 samples, the advantage of TSVD in terms of P_{ratio} is: by 3.7 times for $= 15$ and by 4.2 times for $K = 90$.

Comparison of RP and MUSIC implemented in real numbers, under conditions of correlated sources, and five samples showed RP advantage in terms of P_{ratio} by 3 times for $K = 15$ and by 4.4 times for $K = 90$. At 100 samples, the advantage of RP in terms of P_{ratio} is: by 3.8 times for $K = 15$ and by 4.2 times for $K = 90$.

Methods the DOA estimation based on l_2-regularization, in contrast to those based on l_1-regularization, do not impose restrictions on the properties of the input-output transformation matrix, and also allow efficient hardware implementations. The merits of l_2-regularization include the fact that unlike such methods as MUSIC, it does not require a priori information on the number of signal sources.

Let us consider the following promising directions for further research.

The characteristics of the random projection method for the real-valued case were investigated. It is of interest to study the characteristics of the random projection method for the complex-valued case.

Radar images (2D or 3D) which are formed by processing radio signals emitted by the radar and reflected by the object, have a widespread use in various application areas. For example, in geoinformatics [52, 53] and other areas they use such images obtained by remote sensing of the Earth from space [54–57].

Advantages over optical images include the possibility of obtaining at any time of day, in the presence of cloudiness, under different weather conditions, and with characteristics of objects different from optical images [58, 54–57]. So, they are used for automated mapping of various crops and assessment of the structure of their areas and conditions, forecasting the harvest, etc. [54–57]. In addition, it is possible to display objects under snow, vegetation, and even underground [58].

When receiving radar images, beamforming is used, i.e. antennas with a formed radiation pattern (scanning is performed perpendicular to the direction of motion of the satellite), for which the method proposed in this article can also be applied. Therefore, it is important to study its adaptation and characteristics in the task of obtaining radar images.

A synthetic aperture radar (SAR) produce images by processing the records of the reflected signal of certain part of the Earth's surface from a variety of antenna positions obtained by the motion of its carrier. In this case, taking into account the Doppler shift of the frequencies of the received signals makes it possible to "focus" at a certain point. This gives a large effective virtual ("synthesized") antenna aperture for a particular point and significantly increases the image resolution compared to the physical antenna used [58] (along the direction of motion, the resolution reaches half the length of the physical antenna, regardless of the distance to the object).

The reverse SAR is obtained by observing a moving object from a fixed antenna.

Therefore, it is of interest to study the application of methods based on a stable solution to discrete inverse problem to SAR, by analogy with [59].

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ЗАСТОСУВАННЯ ВИПАДКОВОЇ ПРОЕКЦІЇ ТА УСІЧЕНОГО СИНГУЛЯРНОГО РОЗКЛАДАННЯ ДЛЯ ВИЗНАЧЕННЯ НАПРЯМКУ ПРИХОДУ СИГНАЛІВ ЗА ДОПОМОГИ АНТЕННОГО МАСИВУ

Вступ. Необхідність розв'язання обернених задач виникає в багатьох галузях науки і техніки у зв'язку з відновленням сигналу об'єкта за результатами непрямих дистанційних вимірювань. У разі, коли матриця перетворення має високе число обумовленості, ряд її сингулярних чисел спадає до нуля і вихід вимірювальної системи містить шум, задачу оцінювання вектора входу називають дискретною некоректною оберненою задачею (ДНЗ). Відомо, що рішення ДНЗ з використанням псевдообернення матриці перетворення вхід-вихід є нестійким. Для подолання нестійкості і підвищення точності рішення використовують методи регуляризації.

Розроблені підходи до забезпечення стійкості рішення ДНЗ (усічене сингулярне розкладання TSVD і випадкову проектування RP) використовують цілочисельний параметр регуляризації, в якості якого виступає кількість членів лінійної моделі. Регуляризація з цілочисельним параметром дає можливість забезпечити вибір моделі, близької до найкращої за точністю відновлення вхідного вектора, а також дозволяє знизити обчислювальну складність за рахунок зниження розмірності задачі.

Мета. Розробити підхід до визначення напрямку приходу сигналів в антенну систему за допомоги рішення ДНЗ, провести порівняння результатів з відомим методом MUSIC, виявити переваги та недоліки методів.

Результати. Порівняння TSVD і MUSIC, реалізованих в дійсних числах, під час роботи з корельованими джерелами по п'ятьох зразках показало перевагу TSVD за показником потужності корисного сигналу $P_{\rm ratio}$ в 2.2 раз у разі кількості антенних елементів $K = 15$ і в 4.7 раз у разі $K = 90$, під час роботи по 100 зразках перевагу TSVD за показником $P_{\rm ratio}$ становить 3.7 раз у разі $K = 15$ і 4.2 раз у разі $K = 90$. Порівняння RP і MUSIC, реалізованих в дійсних числах, під час роботи з корельованими джерелами по п'ятьох зразках показало перевагу RP за показником $P_{\rm ratio}$ в 3 рази у разі $K = 15$ і в 4.4 рази у разі $K = 90$, під час роботи по 100 зразках перевагу RP за показником $P_{\rm ratio}$ становить 3.8 раз у разі $K = 15$ і 4.2 раз у разі $K = 90$.

Висновки. Підхід до визначення напрямку приходу сигналів на основі методів l_2-регуляризації забезпечує стійке рішення в разі малої кількості зразків, високого зашумлення та корельованості сигналів джерел. Методи визначення напрямку приходу сигналів на основі l_2-регуляризації, на відміну від l_1-регуляризації, що не накладає обмежень на властивості матриці перетворення вхід-вихід, не вимагають апріорної інформації про кількість джерел сигналу, дозволяють будувати ефективні апаратні реалізації.

Ключові слова: *визначення напрямку приходу сигналів, усічене сингулярне розкладання, випадкова проекція, MUSIC.*

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ПРИМЕНЕНИЕ СЛУЧАЙНОГО ПРОЕЦИРОВАНИЯ И УСЕЧЕННОГО СИНГУЛЯРНОГО РАЗЛОЖЕНИЯ ДЛЯ ОПРЕДЕЛЕНИЯ НАПРАВЛЕНИЯ ПРИХОДА СИГНАЛОВ С ПОМОЩЬЮ АНТЕННОГО МАССИВА

В статье задача определения направления прихода сигналов (НПС) в антенной системе рассматривается как ДНЗ. Приводится линейная модель получения вектора выхода по вектору входа (источников сигналов), рассматривается важность определения НПС по малому количеству векторов выхода (образцов или кадров), дан краткий обзор методов определения НПС. Рассматриваются методы решения ДНЗ на основе усеченного сингулярного разложения и случайного проецирования и их применение для определения НПС. Приводятся результаты имитационного моделирования определения НПС этими методами и сравнение с результатами, полученными известным методом MUSIC.

Ключевые слова: *определение направления прихода сигналов, усеченное сингулярное разложение, случайное проецирование, MUSIC.*